Eur. Phys. J. D **13**, 27–31 (2001)

THE EUROPEAN PHYSICAL JOURNAL D

EDP Sciences

Spontaneous relativistic two-photon decay rate mathematical expression in helium-like systems

J.P. Santos[1,2,a], P. Patte[2], F. Parente[2,3], and P. Indelicato[4]

[1] Departamento de Física, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Monte de Caparica, 2825-114 Caparica, Portugal
[2] Centro de Física Atómica da Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal
[3] Departamento de Física, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Edifício C1, 1749-016 Lisboa, Portugal
[4] Laboratoire Kastler-Brossel[b], École Normale Supérieure et Université Pierre et Marie Curie, Case 74, 4 place Jussieu, 75252 Paris Cedex 05, France

Received 24 March 2000 and Received in final form 19 September 2000

Abstract. We derive a theoretical expression for the two-photon emission rate of two-electron systems, in a form suitable for easy implementation in numerical calculations. Racah algebra techniques were used to extended previous work on two-photon emission in hydrogen-like systems to more complex ones. The obtained expression is, as far as we are aware, the first general expression that gives the spontaneous two-photon decay rates of helium-like systems for any combination of multipoles.

PACS. 31.10.+z Theory of electronic structure, electronic transitions, and chemical binding – 31.30.Jv Relativistic and quantum electrodynamic effects in atoms and molecules – 32.70.Fw Absolute and relative intensities

1 Introduction

Early interest in the decay of metastable states of hydrogen and helium stemmed from astrophysics. Under the low-density conditions that prevail in planetary nebulae, for example, the $1s2s\ ^1\mathrm{S}_0$ state of helium, will depopulate primarily by two-photon emission, while the $1s2s\ ^3\mathrm{S}_1$ state depopulates at approximately equal rates by collisions and by radiation [1]. Consequently, the ratio of line intensities in the helium triplet spectrum can be used as a density probe for the nebula [2,3]. The spectra of helium-like ions in the solar corona and solar flare provide similar information [1] that can lead to the determination of electron densities and temperatures in thermal cosmic X-ray sources [1,4].

Precision lifetime measurements can readily test the theory of atomic structure by providing experimental results that are sensitive to both the wave functions and energies of given configurations.

Although the basic non-relativistic theory of two-photon emission has been available since the beginning of the thirties with the work of Goppert-Mayer [5], laboratory experimentation were not performed before 1965. Lipeles, Novick and Tolk [6] by the application of atomic beam techniques pioneered the study of the $2s \rightarrow 1s$ transition in ionized helium. Their basic method was subsequently enhanced by Marrus and Schmieder [7] who studied a number of heavy hydrogenic and helium-like ions, using the developed beam-foil technology.

Since that time, the increased availability of highly charged heavy ions from accelerator facilities around the world have enabled many experiments, among which Marrus and Schmieder [7] (H-like and He-like argon), Marrus *et al.* [8] (He-like Kr), Dunford *et al.* [9] (H-like and He-like Ni), Dunford *et al.* [10] (He-like Br) and Simionovici *et al.* [11] (He-like Nb).

Accurate knowledge of two-photon rates is also important for helium-like uranium. The $2s$ Lamb-shift was studied by Munger and Gould [12] in a measurement of the $1s2p\ ^3\mathrm{P}_0$ lifetime, which depends on the E1M1 rate of de-excitation to the $1s^2\ ^1\mathrm{S}_0$ ground state for around 15%. The uranium $1s2p\ ^3\mathrm{P}_0$ lifetime is also important for proposed experiments to measure the nuclear magnetic moments of Coulomb-excited nuclear states [13]. The lifetime of the $1s2p\ ^3\mathrm{P}_0$ and $1s2s\ ^1\mathrm{S}_0$ is also important for proposed parity violation experiment in He-like U [14]. The knowledge of the E1M1 contribution to the $1s2p\ ^3\mathrm{P}_0$ lifetime in helium-like gold is also useful for an ongoing experiment at GSI (Darmstadt) in which the hyperfine quenching of this level is measured [15].

[a] e-mail: jps@cii.fc.ul.pt

[b] Unité mixte de recherche du CNRS n° C8552

This work is the continuation of a previous one [16], in which we have studied the hydrogen-like systems, towards the construction of a general, many-electron code in the framework of the multiconfiguration Dirac-Fock method (MCDF) [17–21] where use will be made of B-splines basis sets to compute high-precision values of two-photon spontaneous emission rates in many-electron systems.

2 Two-photon transitions

The basic expression for the two-photon transitions differential emission rate is, in atomic units, [22]

$$\frac{\mathrm{d}w}{\mathrm{d}\omega_1} = \frac{\omega_1\omega_2}{(2\pi)^3c^2}\left|\sum_n \frac{\langle \mathrm{f}|\,\widetilde{A}_2^*\,|n\rangle\,\langle n|\,\widetilde{A}_1^*\,|\mathrm{i}\rangle}{E_n - E_\mathrm{i} + \omega_1} + \frac{\langle \mathrm{f}|\,\widetilde{A}_1^*\,|n\rangle\,\langle n|\,\widetilde{A}_2^*\,|\mathrm{i}\rangle}{E_n - E_\mathrm{i} + \omega_2}\right|^2 \mathrm{d}\Omega_1\mathrm{d}\Omega_2 \quad (1)$$

where i and f denote the initial and final states, ω_j is the frequency, and $\mathrm{d}\Omega_j$ the element of solid angle for the jth photon. The summation over n includes integrations over the continua for both positive and negative energy solutions of the Dirac equation. Conservation of the energy requires

$$E_\mathrm{i} - E_\mathrm{f} = \omega_1 + \omega_2, \quad (2)$$

which permits only one of the two photon frequencies to be independent.

For photon plane waves with propagation vector $\mathbf{k}_j$ and polarization vector $\hat{\mathbf{e}}_j$ ($\hat{\mathbf{e}}_j \cdot \mathbf{k}_\mathrm{f}$=0), the operators $\widetilde{A}_j^*$ in (1) are given by

$$\widetilde{A}_j^* = \boldsymbol{\alpha}\cdot\left(\hat{\mathbf{e}}_j + G\hat{\mathbf{k}}_j\right)\mathrm{e}^{-\mathrm{i}\mathbf{k}_j\cdot\mathbf{r}} - G\mathrm{e}^{-\mathrm{i}\mathbf{k}_j\cdot\mathbf{r}} \quad (3)$$

where G is an arbitrary gauge parameter. From the general requirement of gauge invariance, we would expect the final results to be independent of G. Following the treatment given in [23], the total emission rate, or decay rate, is given by (in atomic units)

$$\frac{\mathrm{d}W}{\mathrm{d}\omega_1} = \frac{\omega_1\omega_2}{(2\pi)^3c^2}\sum_{L_1,M_1,\lambda_1,L_2,M_2,\lambda_2}\left|B_{L_1M_1\lambda_1}^{L_2M_2\lambda_2} + B_{L_2M_2\lambda_2}^{L_1M_1\lambda_1}\right|^2, \quad (4)$$

where

$$B_{L_1M_1\lambda_1}^{L_2M_2\lambda_2} = \sum_n \frac{\left\langle \mathrm{f}\left|\tilde{a}_{L_2M_2}^{\lambda_2}(r)\right|n\right\rangle\left\langle n\left|\tilde{a}_{L_1M_1}^{\lambda_1}\right|\mathrm{i}\right\rangle}{E_n - E_\mathrm{i} + \omega_1}. \quad (5)$$

To obtain equation (5) the partial-wave expansion of the operator $\tilde{A}_j$ was used,

$$\tilde{A}_j = \sum_{\lambda,L,M}\left[\mathbf{e}_j\cdot\mathbf{Y}_{LM}^{\lambda}(\mathbf{k}_j)\right]\tilde{a}_{LM}^{\lambda}(r), \quad (6)$$

where $\mathbf{e}_j$ is an arbitrary polarization vector, $\mathbf{Y}_{LM}^{\lambda}$ are the vector spherical harmonics and λ stands for the electric terms ($\lambda = 1$), magnetic terms ($\lambda = 0$) and the longitudinal terms ($\lambda = -1$).

One of the major task in the calculation of the two-photon emission rates is the derivation of the angular matrix elements in equation (5). In [23] a complete expression for hydrogenic systems was presented, that could be easily used in numerical calculations. In this work we present a derivation of a similar expression for two-electron systems that can be easily used for computing purposes.

This expression will be useful for calculations with two-electron wave functions in the context of Relativistic Configuration Interaction (RCI) or Multiconfiguration Dirac-Fock (MCDF) methods in the frozen core approximation in which the same orthogonal spectator orbitals are used to build both initial and final states.

3 Matrix elements for 2-electron systems

Let us consider an irreducible tensor operator of order K, $\mathbf{X}^{(K)}$, whose components are defined by

$$X_Q^{(K)} \equiv \left\{\mathbf{T}^{(k_1)}\mathbf{U}^{(k_2)}\right\}_Q^{(K)} = \sum_{q_1,q_2} T_{q_1}^{(k_1)}U_{q_2}^{(k_2)}\,\langle k_1q_1k_2q_2|k_1k_2KQ\rangle \quad (7)$$

where $T_{q_1}^{(k_1)}$ and $U_{q_2}^{(k_2)}$ are the components of two tensors operators $\mathbf{T}^{(k_1)}$ and $\mathbf{U}^{(k_2)}$. We assume that $\mathbf{T}^{(k_1)}$ and $\mathbf{U}^{(k_2)}$ operate on parts 1 and 2 of a system, respectively. We are interested in the matrix element

$$\left\langle \gamma j_1j_2JM_J\left|X_Q^{(K)}\right|\gamma' j_1'j_2'J'M_J'\right\rangle \quad (8)$$

where the subscripts 1 and 2 label the angular-momentum quantum numbers of the two parts of the system. Capital letters, like J and M_J, refer to the system total angular momentum quantum numbers. Making use of the Wigner-Eckart theorem, we get

$$\left\langle \gamma j_1j_2JM_J\left|X_Q^{(K)}\right|\gamma' j_1'j_2'J'M_J'\right\rangle = (-1)^{J-M_j}\begin{pmatrix} J & K & J' \\ -M_J & Q & M_J' \end{pmatrix}\left\langle \gamma j_1j_2J\left|\left|X^{(K)}\right|\right|\gamma j_1'j_2'J'\right\rangle. \quad (9)$$

Considering that we want to allow $\mathbf{T}^{(k_1)}$ to act on part 1 and $\mathbf{U}^{(k_2)}$ on part 2, we get [24]

$$\left\langle \gamma j_1j_2J\left|\left|X^{(K)}\right|\right|\gamma j_1'j_2'J'\right\rangle = \left\langle \gamma j_1j_2J\left|\left|T^{(k_1)}U^{(k_2)}\right|\right|\gamma' j_1'j_2'J'\right\rangle = \sum_{\gamma''}\left[J,K,J'\right]^{\frac{1}{2}}\begin{Bmatrix} j_1 & j_1' & k_1 \\ j_2 & j_2' & k_2 \\ J & J' & K \end{Bmatrix} \times\left\langle \gamma j_1\left|\left|T^{(k_1)}\right|\right|\gamma'' j_1'\right\rangle\left\langle \gamma'' j_2\left|\left|U^{(k_2)}\right|\right|\gamma' j_2'\right\rangle. \quad (10)$$

$$\left\langle \mathrm{f}\left|\tilde{a}^{\lambda_2}_{L_2M_2}\right|n\right\rangle\left\langle n\left|\tilde{a}^{\lambda_1}_{L_1M_1}\right|\mathrm{i}\right\rangle=(-1)^{J_\mathrm{f}+J_\mathrm{i}+j_{1_\mathrm{f}}+j_{1_n}+2j_2-M_\mathrm{f}-M_n+L_1+L_2}\left[J_\mathrm{f},J_\mathrm{i}\right]^{1/2}\left[J_n\right]$$
$$\times\begin{Bmatrix} J_\mathrm{f} & L_2 & J_n \\ j_{1_n} & j_{2_\mathrm{f}} & j_{1_\mathrm{f}}\end{Bmatrix}\begin{pmatrix} J_\mathrm{f} & L_2 & J_n \\ -M_\mathrm{f} & M_2 & M_n\end{pmatrix}\begin{Bmatrix} J_n & L_1 & J_\mathrm{i} \\ j_{1_\mathrm{i}} & j_{2_n} & j_{1_n}\end{Bmatrix}\begin{pmatrix} J_n & L_1 & J_\mathrm{i} \\ -M_n & M_1 & M_\mathrm{i}\end{pmatrix}$$
$$\times\left\langle\gamma_\mathrm{f}j_{1_\mathrm{f}}\left\|\tilde{a}^{\lambda_2}_{L_2M_2}\right\|\gamma_nj_{1_n}\right\rangle\left\langle\gamma_nj_{1n}\left\|\tilde{a}^{\lambda_1}_{L_1M_1}\right\|\gamma_\mathrm{i}j_{1_\mathrm{i}}\right\rangle\delta(\gamma_{2_\mathrm{f}},\gamma_{2_n})\delta(\gamma_{2_n},\gamma_{2_\mathrm{i}})\delta(j_{2_\mathrm{f}},j_{2_n})\delta(j_{2_n},j_{2_\mathrm{i}}) \quad (15)$$

$$B^{L_2M_2\lambda_2}_{L_1M_1\lambda_1}=\sum_{j_{1_n}j_{2_\mathrm{i}},J_nM_n}(-1)^{J_\mathrm{f}+J_\mathrm{i}-M_\mathrm{f}-M_n+j_{1_\mathrm{f}}+j_{1_n}+2j_2+L_1+L_2}\left[J_\mathrm{f},J_\mathrm{i}\right]^{1/2}\left[J_n\right]$$
$$\times\begin{Bmatrix} J_\mathrm{f} & L_2 & J_n \\ j_{1_n} & j_2 & j_{1_\mathrm{f}}\end{Bmatrix}\begin{pmatrix} J_\mathrm{f} & L_2 & J_n \\ -M_\mathrm{f} & M_2 & M_n\end{pmatrix}\begin{Bmatrix} J_n & L_1 & J_\mathrm{i} \\ j_{1_\mathrm{i}} & j_2 & j_{1n}\end{Bmatrix}\begin{pmatrix} J_n & L_1 & J_\mathrm{i} \\ -M_n & M_1 & M_\mathrm{i}\end{pmatrix}\frac{\left\langle\gamma_\mathrm{f}j_{1_\mathrm{f}}\left\|\tilde{a}^{\lambda_2}_{L_2M_2}\right\|\gamma_nj_{1_n}\right\rangle\left\langle\gamma_nj_{1n}\left\|\tilde{a}^{\lambda_1}_{L_1M_1}\right\|\gamma_\mathrm{i}j_{1_\mathrm{i}}\right\rangle}{E_n-E_\mathrm{i}+\omega_1} \quad (16)$$

Setting

$$k_1=L,\quad k_2=0 \quad (11)$$

we get

$$\left\langle\gamma j_1j_2J\left\|T^{(L)}\right\|\gamma j_1'j_2'J'\right\rangle=\delta(j_2,j_2')\,(-1)^{j_1+j_2+J'+L}\left[J,J'\right]^{1/2}\begin{Bmatrix} J & L & J' \\ j_1' & j_2 & j_1\end{Bmatrix}\times\left\langle\gamma j_1\left\|T^{(L)}\right\|\gamma'j_1'\right\rangle \quad (12)$$

for an operator $\mathbf{T}^{(L)}$ acting only on part 1.

Thus, letting

$$X^{(K)}_Q=\left\{\mathbf{T}^{(L)}\mathbf{1}^{(0)}\right\}^{(L)}_M=T^{(L)}_M, \quad (13)$$

where $\mathbf{1}^{(0)}$ is a unitary tensor operator of order 0, and considering equations (9, 12), we finally obtain

$$\left\langle\gamma j_1j_2JM_J\left|T^{(L)}_M\right|\gamma'j_1'j_2'J'M_J'\right\rangle=(-1)^{J-M_j}\delta(j_2,j_2')\,(-1)^{j_1+j_2+J'+L}\left[J,J'\right]^{1/2}\times\begin{pmatrix} J & L & J' \\ -M_J & L & M_J'\end{pmatrix}\begin{Bmatrix} J & L & J' \\ j_1' & j_2 & j_1\end{Bmatrix}\left\langle\gamma j_1\left\|T^{(L)}\right\|\gamma'j_1'\right\rangle. \quad (14)$$

4 Two-photon transitions in helium-like systems

For He-like systems, *i.e.*, for systems with only two electrons, the matrix elements in equation (5) are

see equation (15) above

where we made use of the expression (14), since the $\tilde{a}^{\lambda}_{LM}$ operators are irreducible tensor operators of order L as the $\mathbf{T}^{(L)}$ operators. j_{α_β} (with $\alpha=1,2$ and $\beta=\mathrm{i},\mathrm{f},n$) identifies the angular momentum of an individual electron.

We now have

see equation (16) above.

Now

$$\left\langle\alpha\left\|\tilde{a}^{(\lambda)}_{LM}\right\|\beta\right\rangle=(-\mathrm{i})^{L+\lambda-1}(-1)^{j_\alpha-1/2}\left(\frac{4\pi}{2L+1}\right)^{1/2}\times\left[j_\alpha,j_\beta\right]^{1/2}\begin{pmatrix} j_\alpha & L & j_\beta \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\overline{M}^{(\lambda,L)}_{\alpha\beta} \quad (17)$$

where $\overline{M}_\mathrm{fi}$ involves only radial integrals [25]. The notation $[j,k,...]$ means $(2j+1)(2k+1)$... For magnetic type multipoles

$$\overline{M}^\mathrm{m}_\mathrm{fi}=\overline{M}^{(0,L)}_\mathrm{fi}=\frac{2L+1}{[L(L+1)]^{1/2}}\left(\kappa_\mathrm{f}+\kappa_\mathrm{i}\right)I^+_L \quad (18)$$

whereas for electric type multipoles the value depends linearly on the gauge parameter

$$\overline{M}^\mathrm{e}_\mathrm{fi}\left(G\right)=\overline{M}^{(1,L)}_\mathrm{fi}+G\overline{M}^{(-1,L)}_\mathrm{fi}, \quad (19)$$

where

$$\overline{M}^{(1,L)}_\mathrm{fi}=\left(\frac{L}{L+1}\right)^{1/2}\left[(\kappa_\mathrm{f}-\kappa_\mathrm{i})I^+_{L+1}+(L+1)I^-_{L+1}\right]-\left(\frac{L+1}{L}\right)^{1/2}\left[(\kappa_\mathrm{f}-\kappa_\mathrm{i})I^+_{L-1}-LI^-_{L-1}\right] \quad (20)$$

$$\overline{M}^{(-1,L)}_\mathrm{fi}=(2L+1)J_L+(\kappa_\mathrm{f}-\kappa_\mathrm{i})\left(I^+_{L+1}+I^+_{L-1}\right)-LI^-_{L-1}+(L+1)I^-_{L+1}. \quad (21)$$

In the notation used by Rosner and Bhalla [26], the $I^\pm_L(\omega)$ and $J_L(\omega)$ integrals are defined as follows:

$$I^\pm_L(\omega)=\int_0^\infty\left(P_\mathrm{f}Q_\mathrm{i}\pm Q_\mathrm{f}P_\mathrm{i}\right)j_L\left(\frac{\omega r}{c}\right)\mathrm{d}r, \quad (22)$$

$$B^{L_2M_2\lambda_2}_{L_1M_1\lambda_1} = (-i)^{L_2+\lambda_2+L_1+\lambda_1-2}(-1)^{J_\mathrm{f}+J_\mathrm{i}+2j_2+L_1+L_2} \times \sum_{j_{1_n}j_{2_\mathrm{i}},J_n}\left[\sum_{1_n} 4\pi \frac{[j_{1n}]^{1/2}[j_{1_\mathrm{f}},j_{1_n},j_{1_\mathrm{i}}]^{1/2}[J_\mathrm{f},J_n,J_\mathrm{i}]^{1/2}}{[L_2,L_1]^{1/2}} \frac{\overline{M}^{(\lambda,L_2)}_{1_\mathrm{f}\gamma_{1_n}}\overline{M}^{(\lambda,L_1)}_{\gamma_{1_n}1_\mathrm{i}}}{E_{\gamma_{1_n}}-E_\mathrm{i}+\omega_1}\begin{Bmatrix} J_\mathrm{f} & L_2 & J_n \\ j_{1_n} & j_2 & j_{1_\mathrm{f}}\end{Bmatrix}\begin{Bmatrix} J_n & L_1 & J_\mathrm{i} \\ j_{1_\mathrm{i}} & j_2 & j_{1n}\end{Bmatrix}\right.$$
$$\left.\times\begin{pmatrix} j_{1_\mathrm{f}} & L_2 & j_{1_n} \\ 1/2 & 0 & -1/2\end{pmatrix}\begin{pmatrix} j_{1n} & L_1 & j_{1_\mathrm{i}} \\ 1/2 & 0 & -1/2\end{pmatrix}\right][J_n]^{1/2}\sum_{M_n}(-1)^{M_n+M_\mathrm{f}+1}\begin{pmatrix} J_\mathrm{f} & L_2 & J_n \\ -M_\mathrm{f} & M_2 & M_n\end{pmatrix}\begin{pmatrix} J_n & L_1 & J_\mathrm{i} \\ -M_n & M_1 & M_\mathrm{i}\end{pmatrix} \quad (24)$$

$$\frac{\mathrm{d}\overline{W}}{\mathrm{d}\omega_1} = \frac{\omega_1\omega_2}{(2\pi)^3c^2[J_\mathrm{i}]}\sum_{L_1,M_1,\lambda_1,L_2,M_2,\lambda_2}\left|B^{L_2M_2\lambda_2}_{L_1M_1\lambda_1}+B^{L_1M_1\lambda_1}_{L_2M_2\lambda_2}\right|^2$$
$$= \frac{\omega_1\omega_2}{(2\pi)^3c^2[J_\mathrm{i}]}\sum_{L_1,M_1,\lambda_1,L_2,M_2,\lambda_2,M_\mathrm{f},M_\mathrm{i}}\left\{\sum_{j_{1_n}j_{2_\mathrm{i}},J_n}\left[S^{J_nj_{1_n}}(2,1)\Theta^{J_n}(2,1)+S^{J_nj_{1_n}}(1,2)\Theta^{J_n}(1,1)\right]\right\}^2 \quad (29)$$

$$\frac{\mathrm{d}\overline{W}}{\mathrm{d}\omega_1} = \frac{\omega_1\omega_2}{(2\pi)^3c^2[J_\mathrm{i}]}\sum_{L_1,\lambda_1,L_2,\lambda_2}\sum_{j_{1_n}j_{2_\mathrm{i}},J_n}\left\{\left[S^{J_nj_{1_n}}(2,1)\right]^2+\left[S^{J_nj_{1_n}}(1,2)\right]^2\right.$$
$$\left.+2\sum_{j'_{1_n}j_{2_\mathrm{i}},J'_n}\left[[J_n,J'_n]^{1/2}(-1)^{2J'_n+L_1+L_2}\begin{Bmatrix} J_\mathrm{f} & J'_n & L_1 \\ J_\mathrm{i} & J_n & L_2\end{Bmatrix}S^{J_nj_{1_n}}(2,1)S^{J'_nj'_{1_n}}(1,2)\right]\right\}. \quad (32)$$

---

and

$$J_L(\omega) = \int_0^\infty (P_\mathrm{f}P_\mathrm{i}+Q_\mathrm{f}Q_\mathrm{i})\, j_L\left(\frac{\omega r}{c}\right)\mathrm{d}r, \quad (23)$$

where P and Q are the large and small components of the radial Dirac wave function, respectively. The photon frequency $(E_\mathrm{i}-E_\mathrm{f})$ is denoted by ω; j_i, κ_i, and E_i are, respectively, the total angular momentum, relativistic number, and energy of the initial state. The corresponding quantities for the final state are j_f, κ_f, and E_f.

Replacing (17) in (16), and using the fact that $2(j_{1_\mathrm{f}}+j_{1_n})$ is even, it follows

see equation (24) above.

Defining

$$S^{J_nj_{1_n}}(2,1) = \sum_{n_j}\frac{\overline{M}^{(\lambda_2,L_2)}_{\mathrm{f},n_j}(\omega_2)\,\overline{M}^{(\lambda_1,L1)}_{\mathrm{i},n_\mathrm{i}}(\omega_1)}{E_{n_j}-E_\mathrm{i}+\omega_1}\Delta^{J_nj_{1_n}}(2,1), \quad (25)$$

where

$$\Delta^{J_nj_{1_n}}(2,1) = \frac{4\pi[j_{1_\mathrm{f}},j_{1_n},j_{1_\mathrm{i}}]^{1/2}}{[L_2,L_1]^{1/2}}\begin{pmatrix} j_{1_\mathrm{f}} & L_2 & j_{1_n} \\ 1/2 & 0 & -1/2\end{pmatrix}\begin{pmatrix} j_{1n} & L_1 & j_{1_\mathrm{i}} \\ 1/2 & 0 & -1/2\end{pmatrix} \times\left[[j_{1n}]^{1/2}[J_\mathrm{f},J_n,J_\mathrm{i}]^{1/2}\begin{Bmatrix} J_\mathrm{f} & L_2 & J_n \\ j_{1_n} & j_2 & j_{1_\mathrm{f}}\end{Bmatrix}\begin{Bmatrix} J_n & L_1 & J_\mathrm{i} \\ j_{1_\mathrm{i}} & j_2 & j_{1n}\end{Bmatrix}\right] \quad (26)$$

and

$$\Theta^{J_n}(2,1) = [J_n]^{1/2}\sum_{M_n}(-1)^{M_n+M_\mathrm{f}+1} \times\begin{pmatrix} J_\mathrm{f} & L_2 & J_n \\ -M_\mathrm{f} & M_2 & M_n\end{pmatrix}\begin{pmatrix} J_n & L_1 & J_\mathrm{i} \\ -M_n & M_1 & M_\mathrm{i}\end{pmatrix}, \quad (27)$$

we obtain

$$B^{L_2M_2\lambda_2}_{L_1M_1\lambda_1} = (-i)^{L_2+\lambda_2+L_1+\lambda_1-2}(-1)^{J_\mathrm{f}+J_\mathrm{i}+2j_2+L_1+L_2-1} \times\sum_{j_{1_n}j_{2_\mathrm{i}},J_n}S^{J_nj_{1_n}}(2,1)\Theta^{J_n}(2,1). \quad (28)$$

Making use of (28), the decay rate, summed over M_f and averaged over M_i, is given by

see equation (29) above.

Using the sum rules

$$\sum_{M_1,M_2,M_\mathrm{f},M_\mathrm{i}}\Theta^{J}(2,1)\Theta^{J'}(2,1) = \delta(J,J') \quad (30)$$

and

$$\sum_{M_1,M_2,M_\mathrm{f},M_\mathrm{i}}\Theta^{J}(2,1)\Theta^{J'}(1,2) = [J,J']^{1/2}(-1)^{2J'+L_1+L_2}\begin{Bmatrix} J_\mathrm{f} & J' & L_1 \\ J_\mathrm{i} & J & L_2\end{Bmatrix} \quad (31)$$

we arrive at the final expression for the decay rate in helium-like systems

see equation (32) above.

5 Conclusions

Expression (32) corresponds to the one obtained by Goldman and Drake [23] for hydrogenic systems. The main differences between these expressions lie in the definition of the radial integral $S^{J_n j_{1_n}}$ (25), namely the angular coefficients $\Delta^{J_n j_{1_n}}$ (25) and Θ^{J_n} (26), and the existence of two extra summations over j_{1_n} and j_{2_i} in the final expression. This expression is, as far as we are aware, the first general expression that gives the spontaneous two-photon decay rates of helium-like systems for any combination of multipoles. This is a great advantage over the expressions obtained by Drake [27] and by Derevianko and Johnson [28], which apply only to two E1 photons transitions.

Furthermore, by a straightforward generalization of expression (14) a spontaneous two-photon decay rates general expression for a N-electron system could be achieved.

This research was supported in part by FCT (Portugal) under project PRAXIS/C/FIS/10030/98, and by the TMR Network Eurotraps Contract Number ERBFMRXCT970144.

References

1. J. Shapiro, G. Breit, Phys. Rev. **113**, 179 (1959).
2. G.W.F. Drake, R.R. Robbins, Astrophys. J. **171**, 55 (1972).
3. R.R. Robbins, Astrophys. J. **160**, 519 (1970).
4. G.R. Blumenthal, G.W.F. Drake, W.H. Tucker, Astrophys. J. **172**, 205 (1972).
5. M. Goppert-Mayer, Ann. Phys. (Leipzig) **9**, 273 (1931).
6. M. Lipeles, R. Novick, N. Tolk, Phys. Rev. Lett. **15**, 690 (1965).
7. R. Marrus, R.W. Schmieder, Phys. Rev. A **5**, 1160 (1972).
8. R. Marrus *et al.*, Phys. Rev. Lett. **56**, 1683 (1986).
9. R.W. Dunford *et al.*, Phys. Rev. Lett. **62**, 2809 (1989).
10. R.W. Dunford *et al.*, Phys. Rev. A **48**, 1929 (1993).
11. A. Simionovici *et al.*, Phys. Rev. A **48**, 1695 (1993).
12. C.T. Munger, H. Gould, Phys. Rev. Lett. **57**, 2927 (1986).
13. L.N. Labzowsky *et al.*, Phys. Rev. Lett. **84**, 851 (2000).
14. A. Schfer *et al.*, Phys. Rev. A **40**, 7362 (1989).
15. P. Indelicato *et al.* (unpublished).
16. J.P. Santos, F. Parente, P. Indelicato, Eur. Phys. J. D **3**, 43 (1998).
17. J.P. Desclaux, Comp. Phys. Commun. **9**, 31 (1975).
18. P. Indelicato, F. Parente, R. Marrus, Phys. Rev. A **40**, 3505 (1989).
19. P. Indelicato, O. Gorceix, J.P. Desclaux, J. Phys. B **20**, 651 (1987).
20. P. Indelicato, Nucl. Instrum. Meth. Phys. B **31**, 14 (1988).
21. O. Gorceix, P. Indelicato, J.P. Desclaux, J. Phys. B **20**, 639 (1987).
22. A.I. Akhiezer, V.B. Berestetskii, *Quantum Electrodynamics* (Interscience Publishers, New York, 1965).
23. S.P. Goldman, G.W.F. Drake, Phys. Rev. A **24**, 183 (1981).
24. B.R. Judd, *Operator Techniques in Atomic Spectroscopy* (Princeton University Press, Princeton, 1998).
25. I.P. Grant, J. Phys. B **7**, 1458 (1974).
26. H.R. Rosner, C.P. Bhalla, Z. Phys. **231**, 347 (1970).
27. G.W.F. Drake, Phys. Rev. A **34**, 2871 (1986).
28. A. Derevianko, W.R. Johnson, Phys. Rev. A **56**, 1288 (1997).